Industrial and investment analysis as a tool for the regulation of public services

Turin 7 September 2016

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Outline

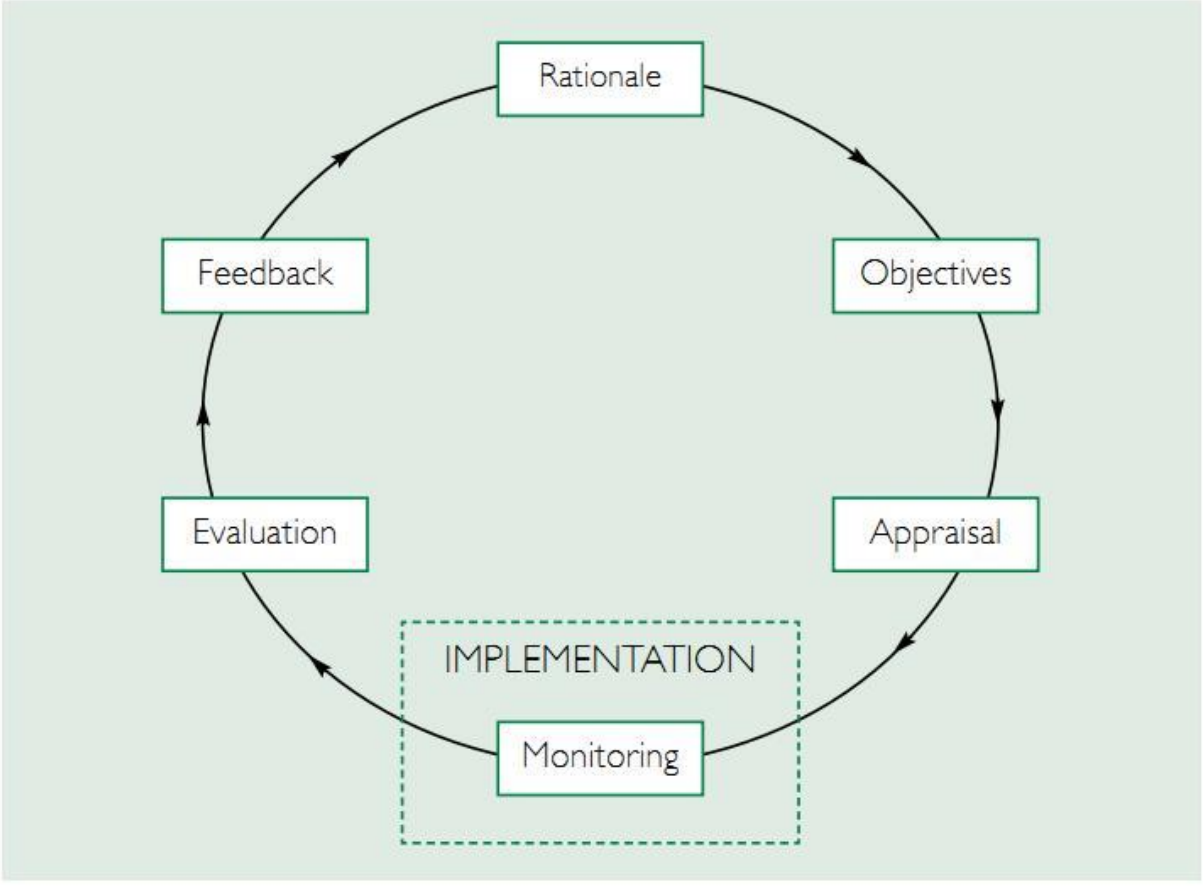
- Examine investment appraisal and the related techniques;
- Discuss the practical application of investment appraisal techniques to the public sector;
- Run through the spreadsheet you will need to use in your working groups next week.

Investment – planting your seeds

Investment – growing your plants

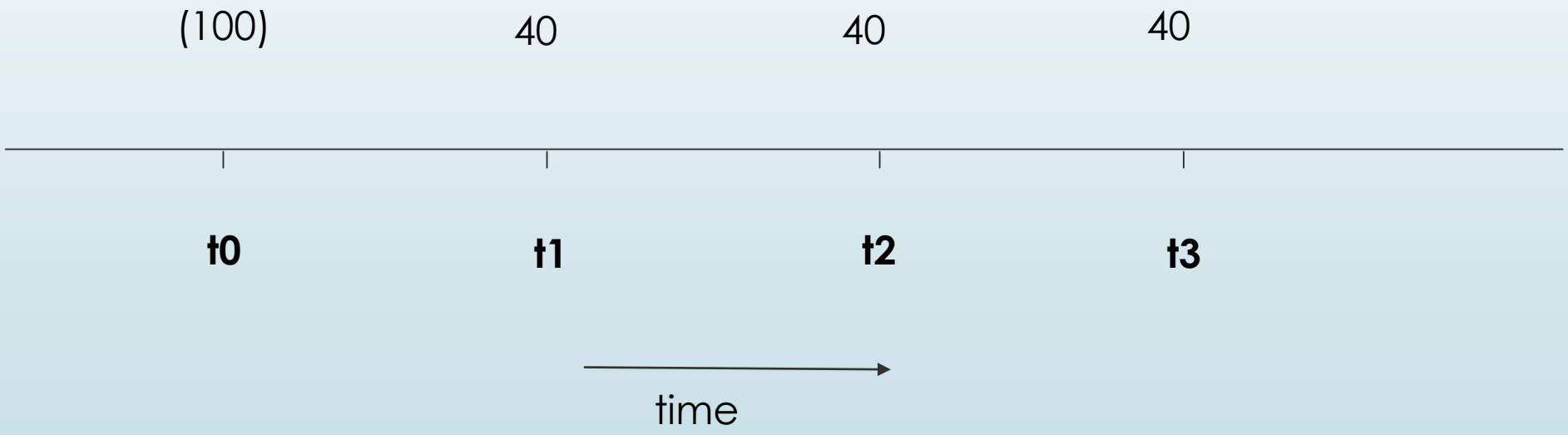
Investment choices

Appraisal and evaluation cycle ROAMEF – Green Book HM Treasury, UK



What is an investment?

It is an outflow/s of cash that is/are expected to lead to future inflow/s of cash.



What is investment appraisal?

- An evaluation **at t0** (now) of the attractiveness of an investment proposal/s.

- The outflow/s and inflow/s of cash occur at **different times**.
- Decision making involves the **future**;
- What has happened in the **past** cannot be altered;
- The future is unpredictable –

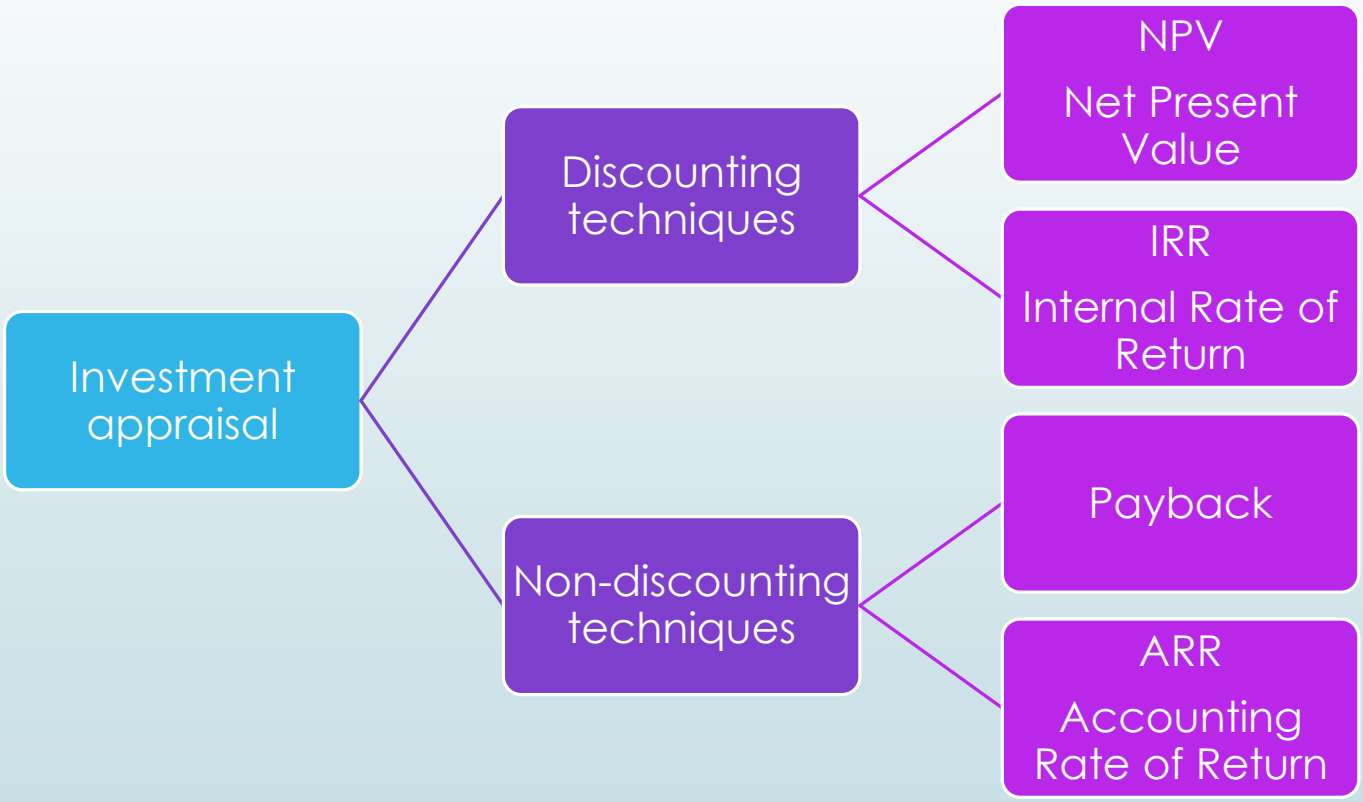
RISK

is an essential part of financial decision making

Why do we need investment appraisal?

- Investment appraisal is useful to determine which projects should be invested in and which should be avoided or postponed.
- In the private sector, the objective of investment appraisal is **the maximisation of shareholders wealth**
- Utilities tend to be capital-intensive, i.e. the requirement for significant plant investment increases the need to raise capital to finance it.
- It can be used in order to obtain (or increase) financing as it can show investors what the expected returns are on an investment project.

Investment appraisal techniques



Risk and return

- Risk and **return** are related: investors require a minimum return to invest, but they require a higher rate of return (a **risk premium**) to compensate them for taking increased risks.

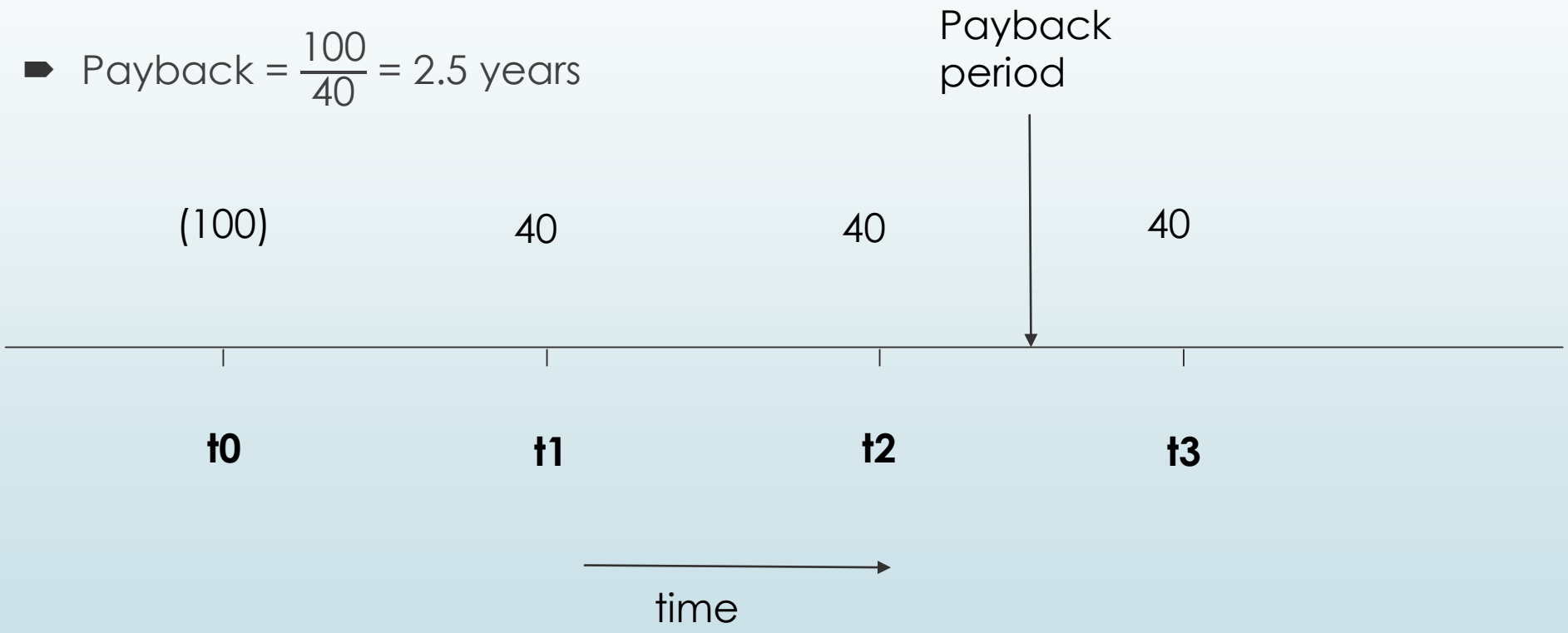
Non – discounting technique PAYBACK

- This technique determines how many years it takes for the cash inflow/s from an investment to pay back the initial outflow/s
- If the inflows are constant then:

$$\text{Payback period in years (PBP)} = \frac{\text{initial investment}}{\text{annual cash inflows}}$$

Non – discounting technique PAYBACK

- Payback = $\frac{100}{40}$ = 2.5 years



Non – discounting technique PAYBACK

- Investor determines a **target period**, i.e. the period by which the investor would like his cash back.
- If payback period is less than target period then investor will accept the project
- If payback period is more than target period then investor will reject the project

Non – discounting technique
PAYBACK advantages and disadvantages

- Advantages:

Easy to calculate and understand

Focuses on earlier cash flows which are more certain in a project's lifetime

- Disadvantages:

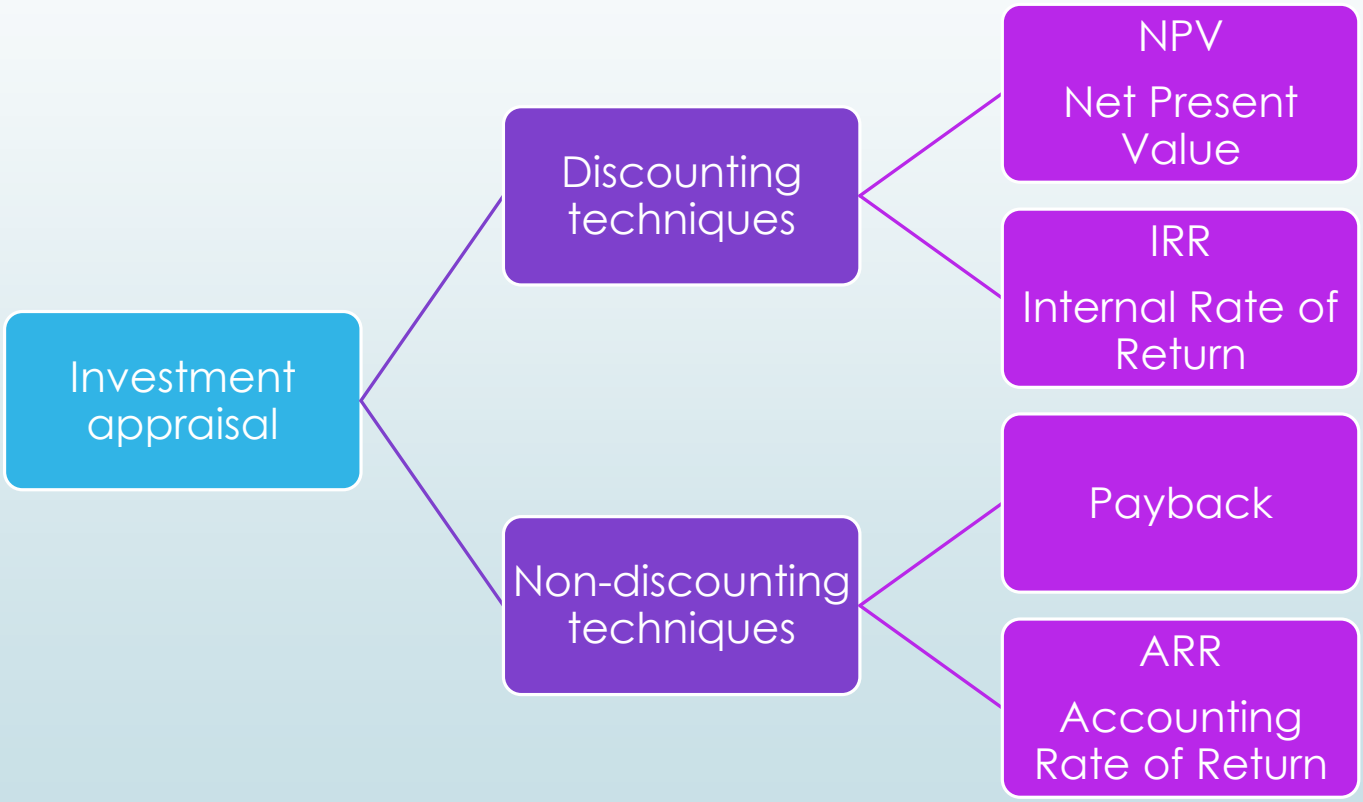
Ignores change in wealth

Ignores cash flows after payback period

Requires a target payback period to be set

Ignores the time value of money

Investment appraisal techniques



Non – discounting technique
ARR (ACCOUNTING RATE OF RETURN)

- It expresses the profits of a project as a percentage of the capital investment.
- It is calculated as:

$$\text{ARR} = \frac{\text{average annual accounting profit}}{\text{initial investment}} \times 100\%$$

- If the ARR is greater than the target rate, the investor will accept the project
- If the ARR is less than the target rate, the investor will refuse the project

Non – discounting technique
ARR advantages and disadvantages

- Advantages:

Easy to calculate and understand

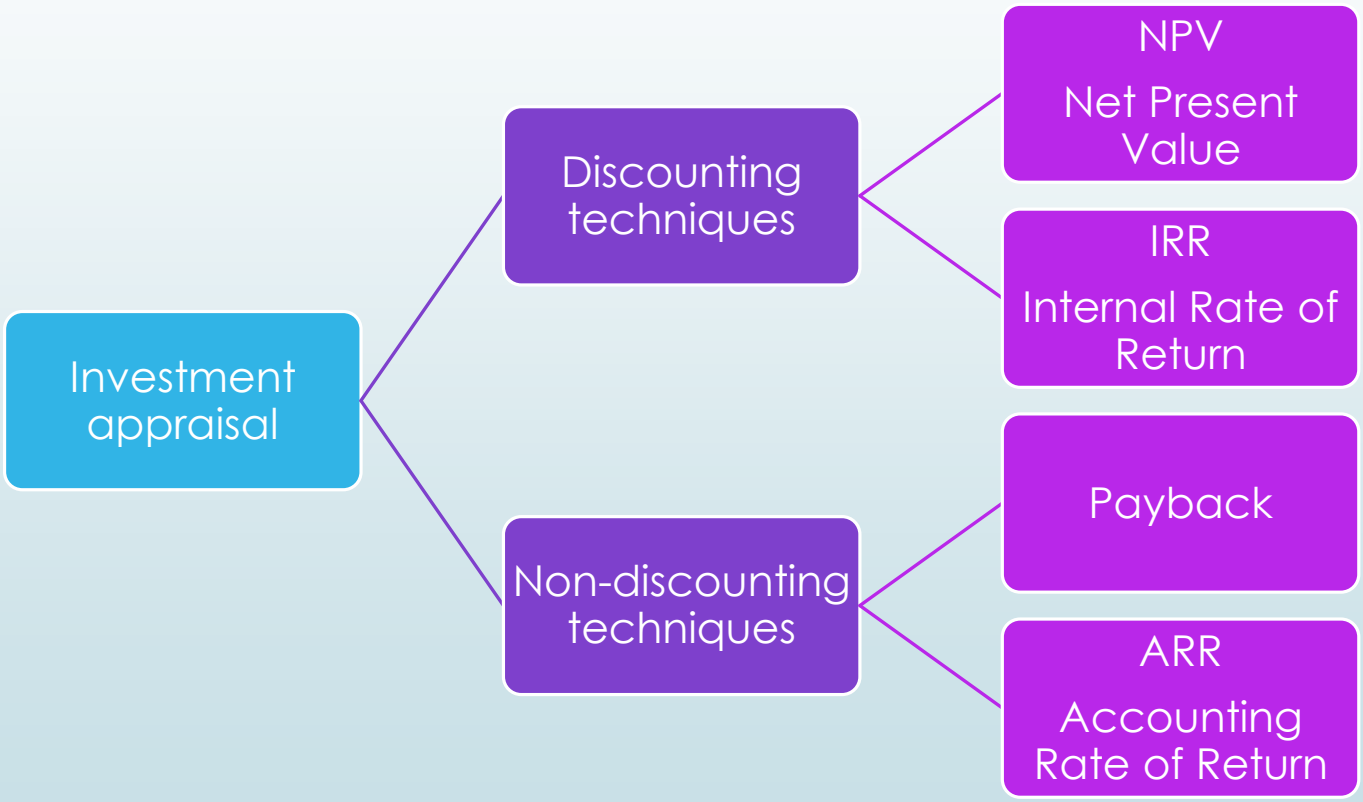
Used by financial analysts to appraise performance

- Disadvantages:

It is based on profits rather than cash flows. (Profits are more subjective, can change depending on the accounting policy and include a number of irrelevant items, such as sunk costs, depreciation, fixed overheads, etc.)

Ignores the time value of money

Investment appraisal techniques



Discounting technique
NPV (Net Present Value)

- It expresses the current value of the cashflows relating to an investment
- It is calculated as the sum of the present value of current and future cash outflows and inflows related to an investment.

- NPV=$\sum_{t=0}^{N} \frac{CF_t}{(1+k)^t}$

- where:

CF= cash flow (inflow or outflow)

t= time period

k= cost of capital

Discounting technique
NPV (Net Present Value)

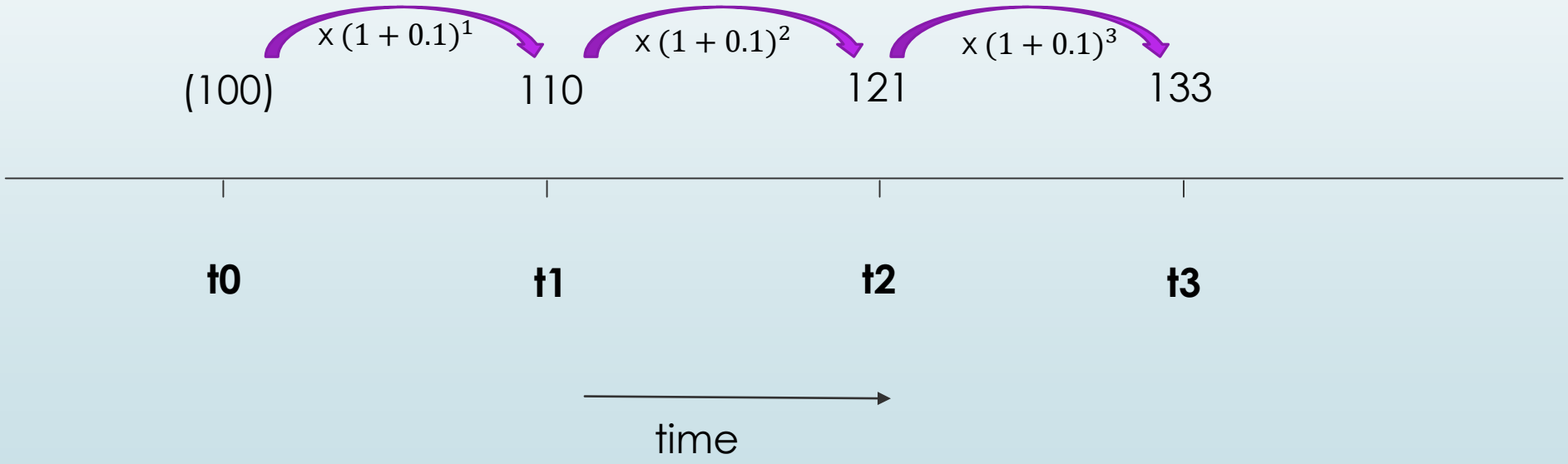
- NPV=$\sum_{t=0}^{N} \frac{CF_t}{(1+k)^t}$

- If NPV is positive, then investment project should be accepted
- If NPV is negative, then investment project should be refused

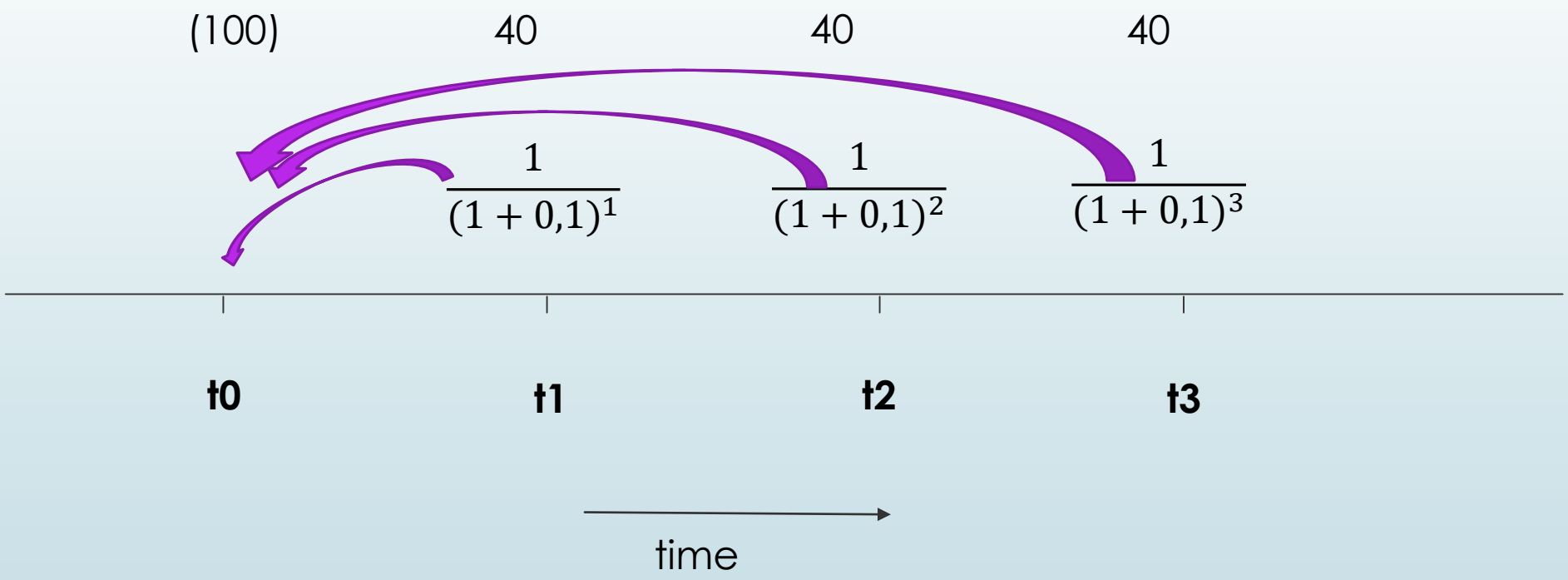
- The higher the NPV, the better the investment project is.

Understanding NPV

If I lend €100 today (t0) and interest is 10% per annum (p.a.), I expect to receive €110 at t1; €121 at t2 or €133 at t3



Net present value -cash flow calculation



Net present value -cash flow calculation

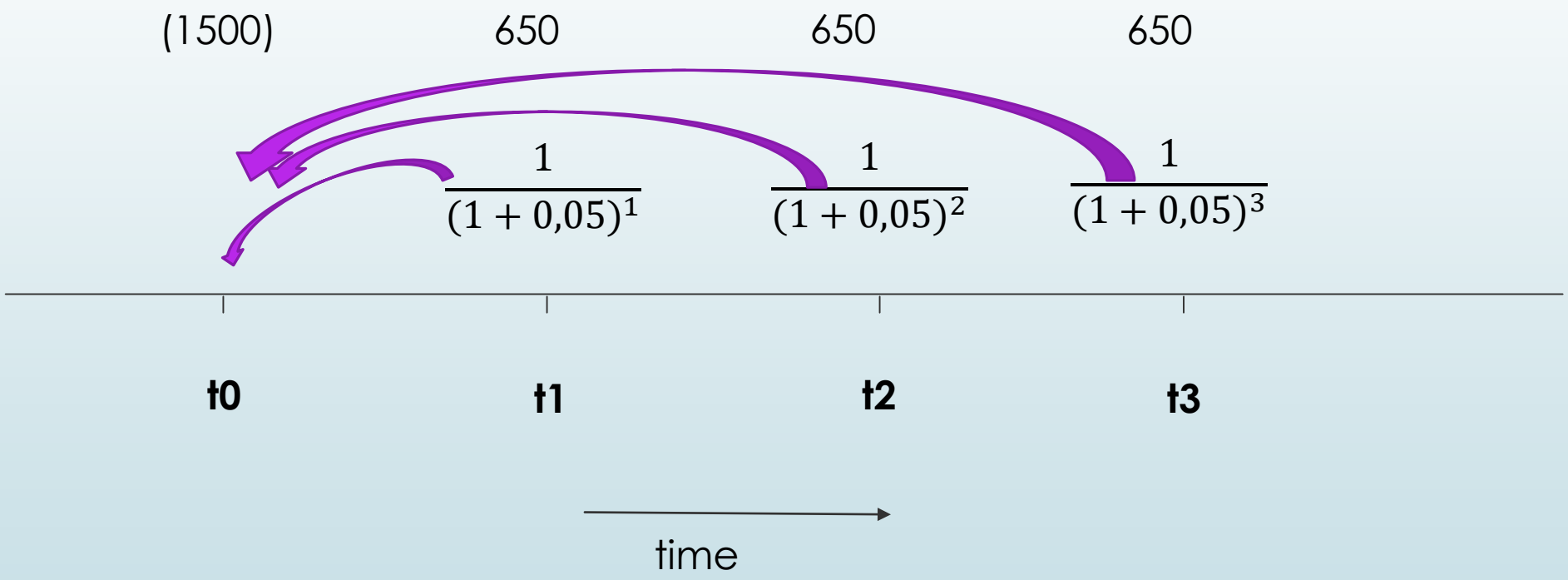
- NPV=$\sum_{t=0}^{N} \frac{CF_t}{(1+k)^t}$

- NPV= $\frac{CF_0}{(1+k)^0} + \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \frac{CF_3}{(1+k)^3}$
- NPV= $\frac{-100}{(1+0.1)^0} + \frac{40}{(1+0.1)^1} + \frac{40}{(1+0.1)^2} + \frac{40}{(1+0.1)^3}$ = -100 +36.36 +33.04+30.04= -0.56
- NPV is negative, investment should be rejected

Net present value – class exercise

- A water utility is considering an investment project for a new water treatment plant.
- The cost of the new treatment plant will be €1500 at t0. The future inflows of cash expected from the project are €650 at t1, t2 and t3.
- If the cost of capital for the company is 5%, calculate the NPV of the project and decide whether the company should go ahead and invest.

Net present value – class exercise solution



Net present value – class exercise solution

- NPV=$\sum_{t=0}^{N} \frac{CF_t}{(1+k)^t}$

- NPV= $\frac{CF_0}{(1+k)^0} + \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \frac{CF_3}{(1+k)^3}$
- NPV= $\frac{-1500}{(1+0.05)^0} + \frac{650}{(1+0.05)^1} + \frac{650}{(1+0.05)^2} + \frac{650}{(1+0.05)^3}$ = -1500 + 618.80 + 589.55 + 561.60 = 269.95

Essential points for NPV calculation

- NPV=$\sum_{t=0}^{N} \frac{CF_t}{(1+k)^t}$

Cash flows

Cost of capital

Cash flows

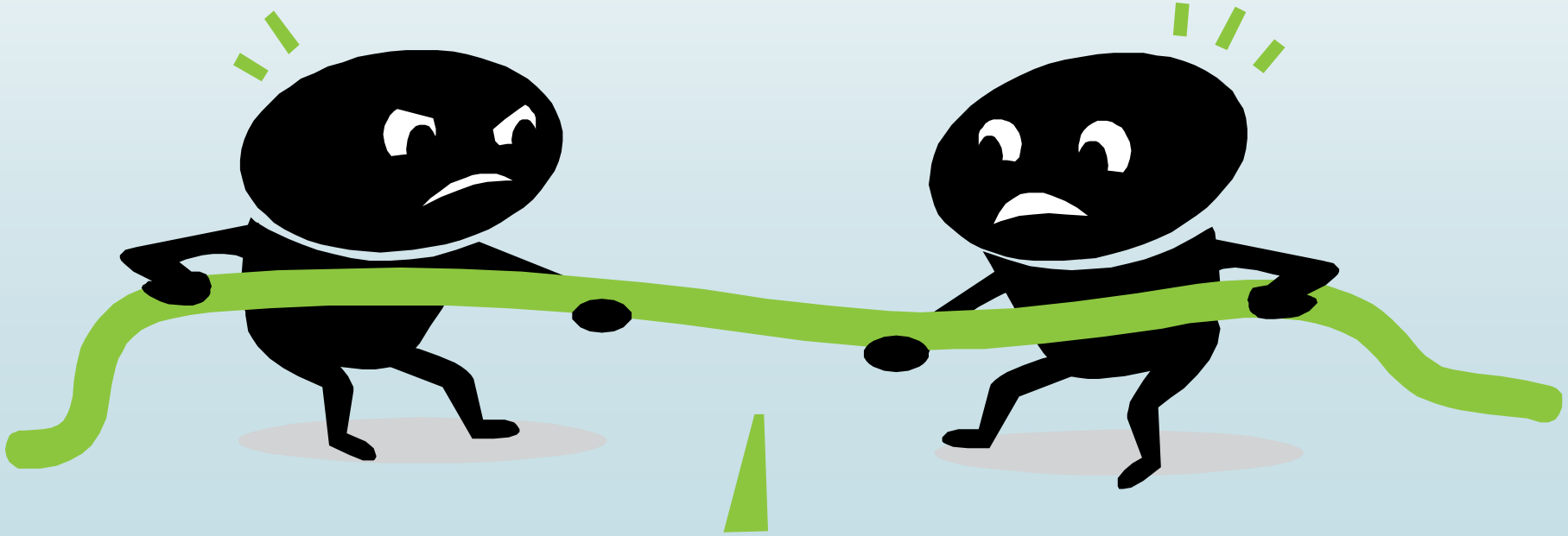
Predicted cash flows MUST be:

- Inflows or outflows of cash, NOT profit

Cash flows

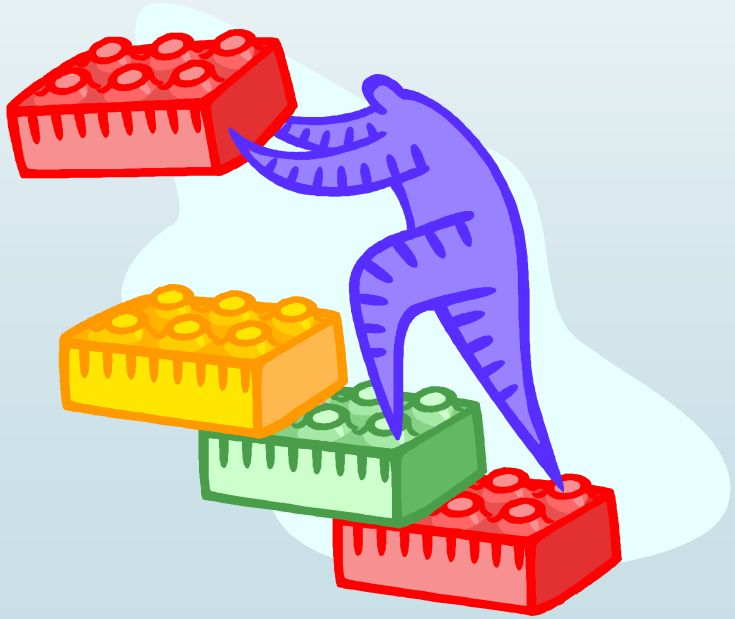
Predicted cash flows MUST be:

- NET = the difference between outflows and inflows in any time period t

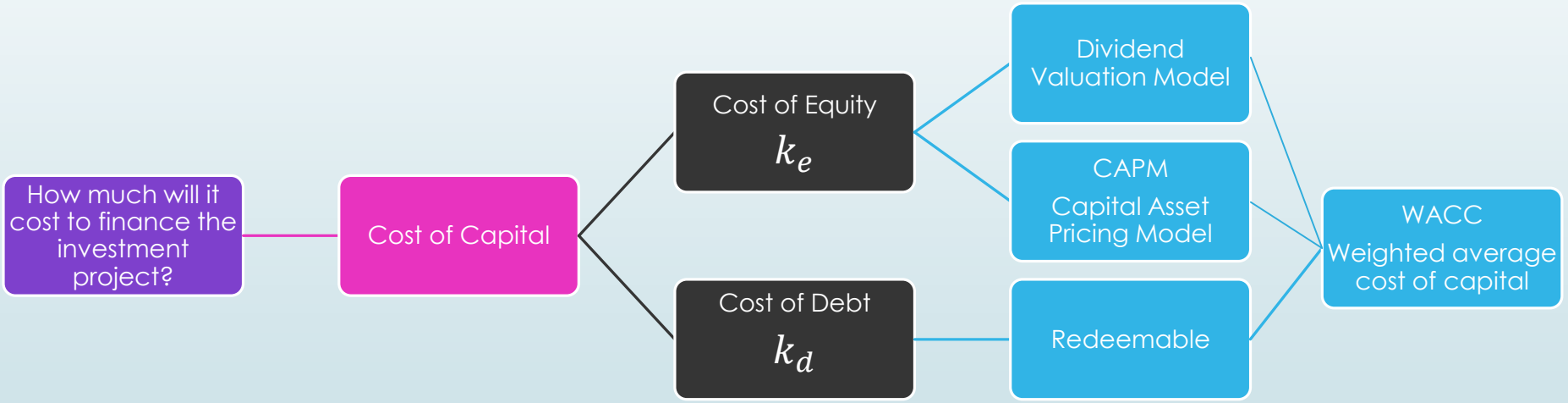


Cash flows

- Differential, i.e. the difference between the new cash flows as a result of the investment and the existing cash flows



Cost of capital – discounting rate



Cost of capital – Cost of Equity Dividend valuation model

- The cost of capital or of each source of long-term finance is the return which the investors expect on their investment.
- In the case of cost of equity it is the future dividend yield expected and capital growth.
- Ideally each future dividend should be measured for each period, however this is difficult and unrealistic so it is assumed that the dividends either remain constant or grow at a fixed annual rate.
- If constant: $k_e = \frac{D_0}{P_0}$
- If growing at constant rate: $k_e = \frac{D_0(1+g)}{P_0} + g$

where:
P_0 = market value of equity
D_0 = dividend just paid
g = constant rate growth

Cost of capital – Cost of Equity Dividend valuation model

- Allows the cost of equity to be calculated using empirical values (available for listed companies)
- BUT does not provide explanation for why different shares have different costs of equity

Cost of capital – Cost of Equity Capital asset pricing model

- CAPM: $k_e = r_f + \beta_j(r_m - r_f)$

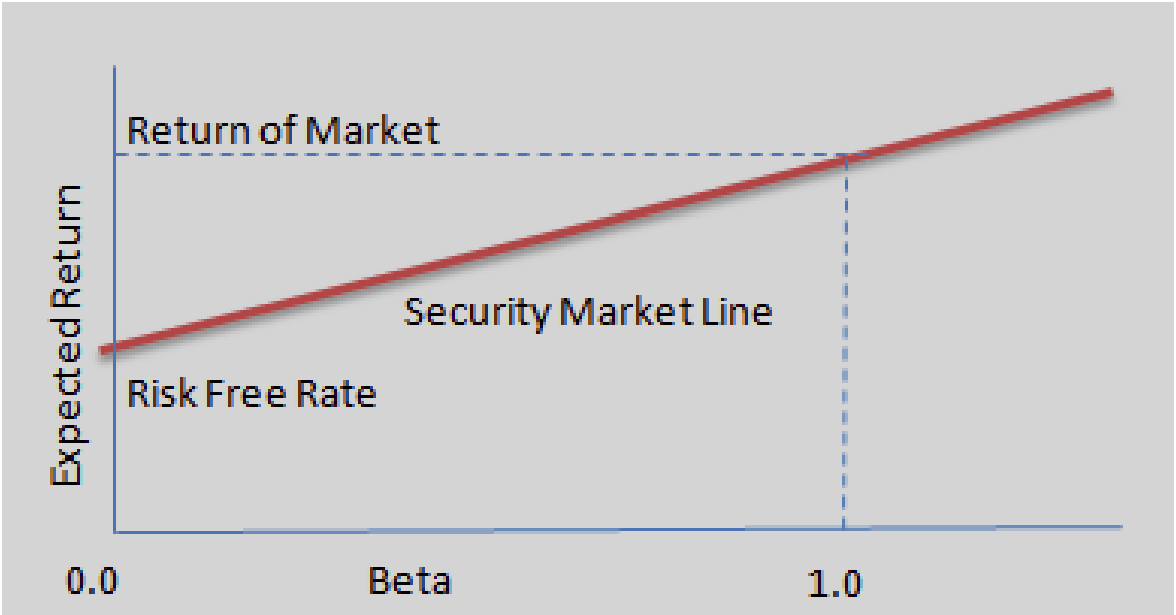
where:
r_f = risk – free rate of interest
r_m = return on the market portfolio
β_j = index of systematic risk for the investment

- $r_m - r_f$ is the market premium. If the market premium is average $\beta = 1$. If the investment project has more systematic risk than market average, β is > 1. If the investment project has less systematic risk than market average, β is < 1

Cost of capital – Cost of Equity Practical solution

- r_f = use Government short-term Treasury bills as these should be a risk-free asset with certain future return.
- $r_m - r_f$ = use the long-term average premium on stock market which is around 5% p.a. as stock market is very volatile and unpredictable.
- β = use the betas of quoted companies in a similar line of business. For public utilities betas are normally <1 as they are frequently price and return regulated. Demand for them is also consistent.

Cost of capital – Cost of Equity CAPM – graphical representation



Cost of capital – Cost of Equity class exercise

- r_f = A UK water utility company based wishes to calculate its cost of equity. The Government short-term Treasury bills (gilts) have a rate of 3%, the return on the market portfolio is 7% and the index of systematic risk is 0.9, calculate the cost of equity for the company?

Cost of capital – Cost of Equity class exercise solution

- A UK water utility company based wishes to calculate its cost of equity. The Government short-term Treasury bills (gilts) have a rate of 3%, the return on the market portfolio is 7% and the index of systematic risk is 0.9, calculate the cost of equity for the company?
- $r_f = 3\%$
- $r_m = 7\%$
- $\beta = 0.9$
- $k_e = r_f + \beta(r_m - r_f) = 0.03 + 0.9\ (0.07\text{-}0.03) = 0.066 = 6.6\%$

Cost of capital – Cost of Debt

- $k_d = \frac{I_0(1-T)}{D_0}$

I_0 = annual interest payment

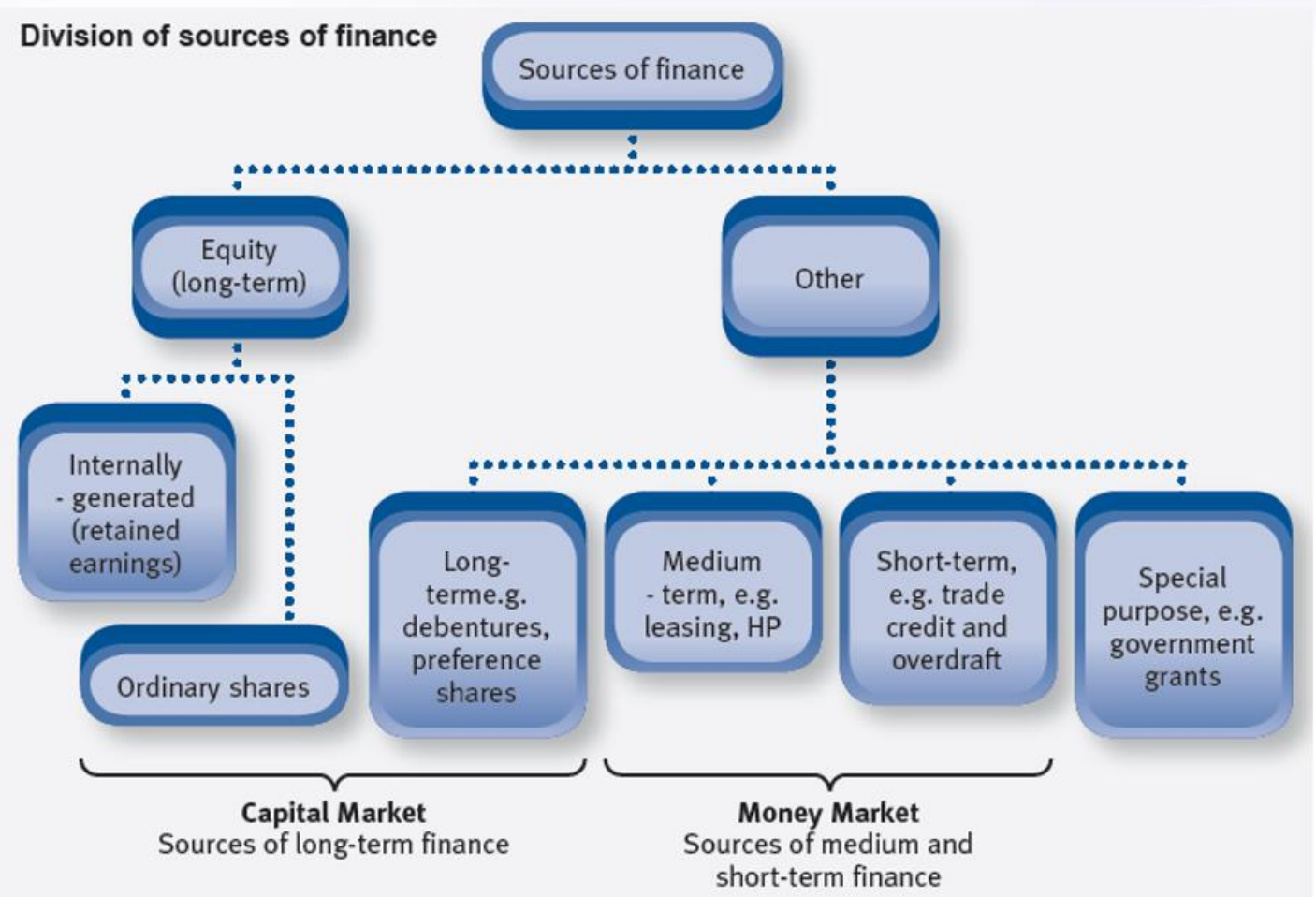
D_0 = amount (market price) of the loan

T = corporation tax rate

Cost of capital – Cost of Debt Practical solution

- The cost of debt can normally be found in the notes to the accounts of the company.
- The corporation tax rate that should be used is that of the country where the utility company is registered to pay tax.

Weighted average cost of capital – sources of finance



Weighted average cost of capital WACC

- The cost of equity and cost of debt of a company must then be combined to determine the weighted average cost of capital of a company.

$$\text{WACC} \quad k = \frac{(MV_e * k_e) + (MV_d * k_d)}{(MV_e + MV_d)}$$

Where

- MV_d is the market value of the debt of the company
- MV_e is the market value of the equity of the company

Weighted average cost of capital – class exercise

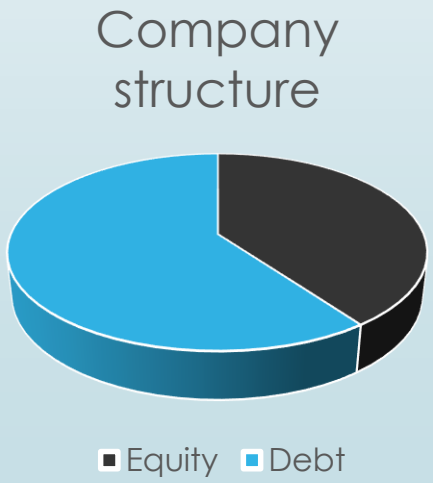
- A gas utility has traditionally raised funds in the proportion 40% equity and 60% debt. The sources of finance have the following costs:

k_e = 10%

k_d = 7%

It is appraising an investment for €1.000.000 in a new gas pipe. What discount rate should the utility use?

WACC $k = \frac{(MV_e * k_e)+(MV_d * k_d)}{(MV_e + MV_d)} = \frac{(40 * 0.1)+(60 * 0.07)}{(100)} = 8.2\%$



WACC – Ofwat price review 09

PR09 inputs	PR09
Real risk-free rate (RFR)	2.0%
Equity market risk premium (EMRP)	5.4%
Asset beta	0.4
Gearing (Debt:RCV)	57.5%
Equity beta	0.9
Cost of equity (post-tax)	7.1%
Cost of debt	3.6%
WACC – gross of tax shield (Vanilla)	**5.1%**
Corporate tax rate (not given in final determination)	28%
WACC – post-tax	**4.5%**

Source: Ofwat

WACC – Ofwat risk and reward guidance PR14

WACC inputs	Ofwat (point estimate)	Ofwat range
Total equity market return	6.75%	6.25% to 6.75%
Real risk-free rate	1.25%	0.75% to 1.25%
Equity market risk premium	5.50%	5.50%
Gearing (Net Debt:RCV)	62.50%	60% to 62.5%
Asset beta	0.3	0.3
Equity beta	0.8	0.75 to 0.80
Cost of equity (post-tax)	5.65%	4.9% to 5.7%
Ratio of embedded debt to new debt	75%:25%	75%:25%
Cost of new debt	2.65%	2.6 to 2.8%
Cost of embedded debt	2.65%	2.6% to 2.8%
Allowance for debt fees	0.1%	0.1%
Overall cost of debt	2.75%	2.7% - 2.9%
Appointee (Vanilla) WACC	**3.85%**	**3.6% to 3.9%**
Adjustment from appointee to wholesale WACC	(0.15)%	(0.15)%
Wholesale WACC	**3.70%**	**3.45% to 3.75%**

Source: Ofwat risk and reward guidance

Discounting techniques
NPV advantages and disadvantages

- Advantages:

It measures in absolute terms the effect of taking on a project now

It considers the time value of money

It is unaffected by accounting policies

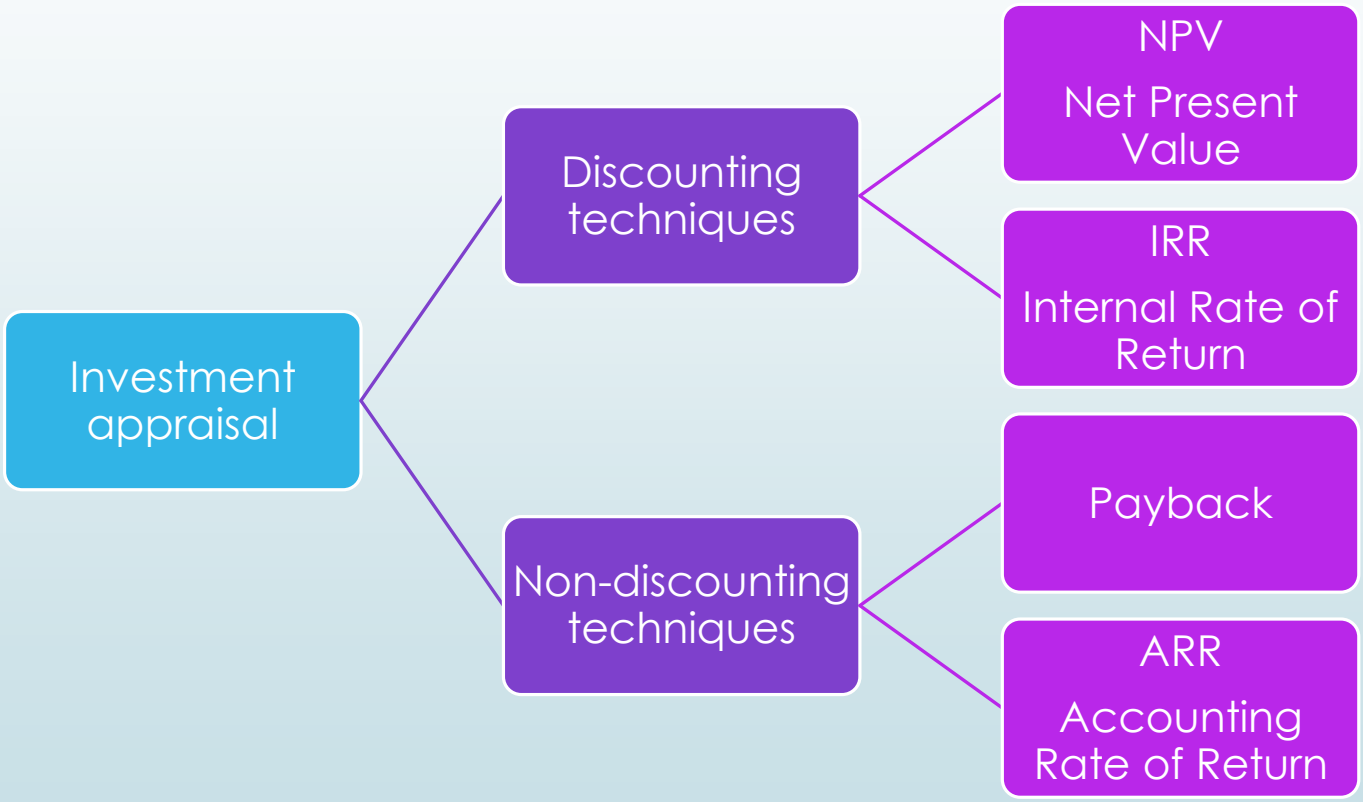
It incorporates risk in the discount rate used to discount the cash-flows

- Disadvantages:

It can be difficult to estimate the discounting rate accurately, i.e. r_m, r_f, β

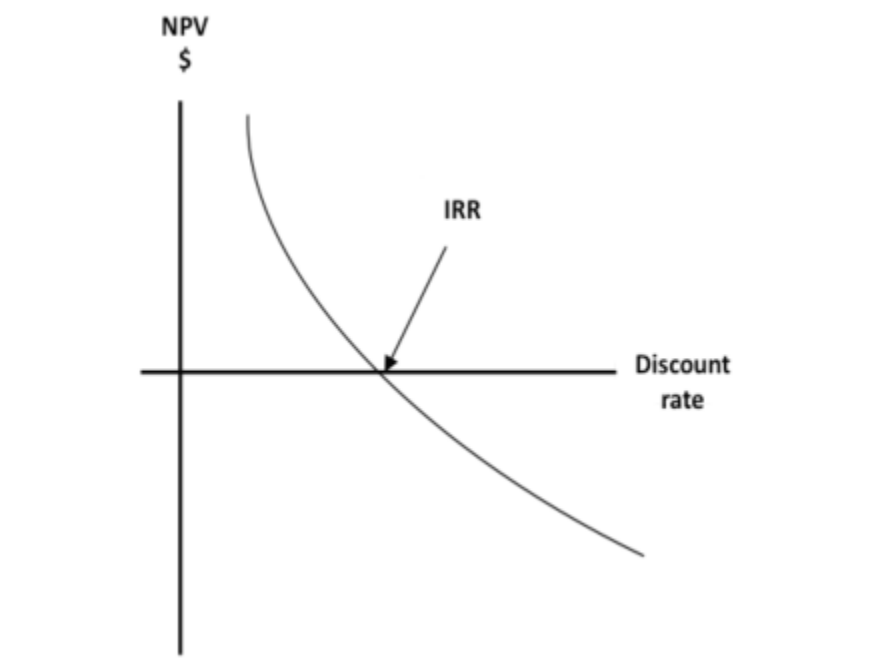
NPV is generally considered the best of the investment appraisal techniques

Investment appraisal techniques



Discounting technique
IRR (Internal Rate of Return)

- It is directly linked to NPV but it is a relative measure rather than an absolute measure.
- It is the discount rate which makes the NPV of the project equal to zero.



Discounting technique
IRR (Internal Rate of Return)

- If the IRR > target discount rate, investment project should be accepted
- If the IRR < target discount rate, investment project should be rejected

- The target discount rate is the company's WACC.

Discounting technique
IRR (Internal Rate of Return)

- Advantages:

It considers the time value of money (as does NPV)

It may be more easily understood than NPV as it is expressed as a %

- Disadvantages:

It is harder to calculate than NPV

If discount rate changes, IRR does not work.

NPV calculation – practical issues

- **From profits to cash flows. Generally the information we have available for companies is the profit and loss account (aka income statement).**
- We therefore need to convert profits to cash flows:

- by adding back **depreciation**
- by adding back **interest**

- by deducting **working capital -**(inventory + receivables – payables) or (current assets – current liabilities)
- by deducting the **cost of the plant** invested in

NPV calculation – proforma

	0	1	2	3	4	5	6
Years	2016	2017	2018	2019	2020	2021	2022
Result for the period	0	0	0	0	0	0	0
Add back							
Depreciation	0	0	0	0	0	0	0
Interest	0	0	0	0	0	0	0
Working capital recovered (last year only)							
Scrap value of asset							
Deduct							
Working capital	0	0	0	0	0	0	0
Cost of asset	0	0	0				
Cash flows	0	0	0	0	0	0	0
Discounted cash flows	0	0	0	0	0	0	0
NPV	0						

NPV – alternative investment projects

- A company may have limited funds and so have to choose between alternative investment projects. If each alternative investment project has the same cash outflow at the beginning (t0) it is the project with the **highest NPV** that will be selected:

	t0	NPV
Project A	(1.000)	212
Project B	(1.000)	184
Project C	(1.000)	(91)

NPV – alternative investment projects

- If the cash outflows at the beginning (t0) are different then it is a little more complex:

	t0	NPV
Project A	(1.000)	212
Project B	(500)	184
Project C	(200)	36

$$\text{Present value index or profitability index} = \frac{\text{Present value of cash inflows}}{\text{Investment required}}$$

NPV – alternative investment projects

- Once the present value index has been calculated, choose the project with the highest index, i.e. project B.

	t0	NPV	PV of cash inflows	Profitability index
Project A	(1.000)	212	1.212	1.20
Project B	(500)	184	684	1.37
Project C	(200)	36	236	1.18

$$\text{Present value index or profitability index} = \frac{\text{Present value of cash inflows}}{\text{Investment required}}$$

NPV – alternative investment projects – divisible/indivisible projects

- However, if the firm has £700 to invest at t0, it will choose project B and A if project A is divisible, it will choose project B and C is project A is indivisible.

	t0	NPV	PV of cash inflows	Profitability index
Project A	(1.000)	212	1.212	1.20
Project B	(500)	184	684	1.37
Project C	(200)	36	236	1.18

$$\text{Present value index or profitability index} = \frac{\text{Present value of cash inflows}}{\text{Investment required}}$$

Applying investment appraisal techniques to public utilities

- Objective of an investment appraisal in a **private sector** company is to **maximise shareholder wealth**.
- In the case of a **public sector** company where instead of shareholders, there is a government, the objective is to **maximise citizens' benefit**.
- **BUT** in public sector investments, the **benefit** to citizens is **disproportionate** (as it is with shareholders in the private sector), i.e. is enjoyed by a small proportion of the community relative to the contribution that the citizens have made to this investment (through taxes paid to the government).
- For example, a new public water treatment plant built in a new area of a developing city, will benefit the people who live in that area but all the taxpayers of that city will have contributed to it despite the fact they are not benefiting from it.

Investment appraisal in public sector - disproportionate

Pay taxes to help build hospital

Benefit from hospital

Applying investment appraisal techniques to public utilities

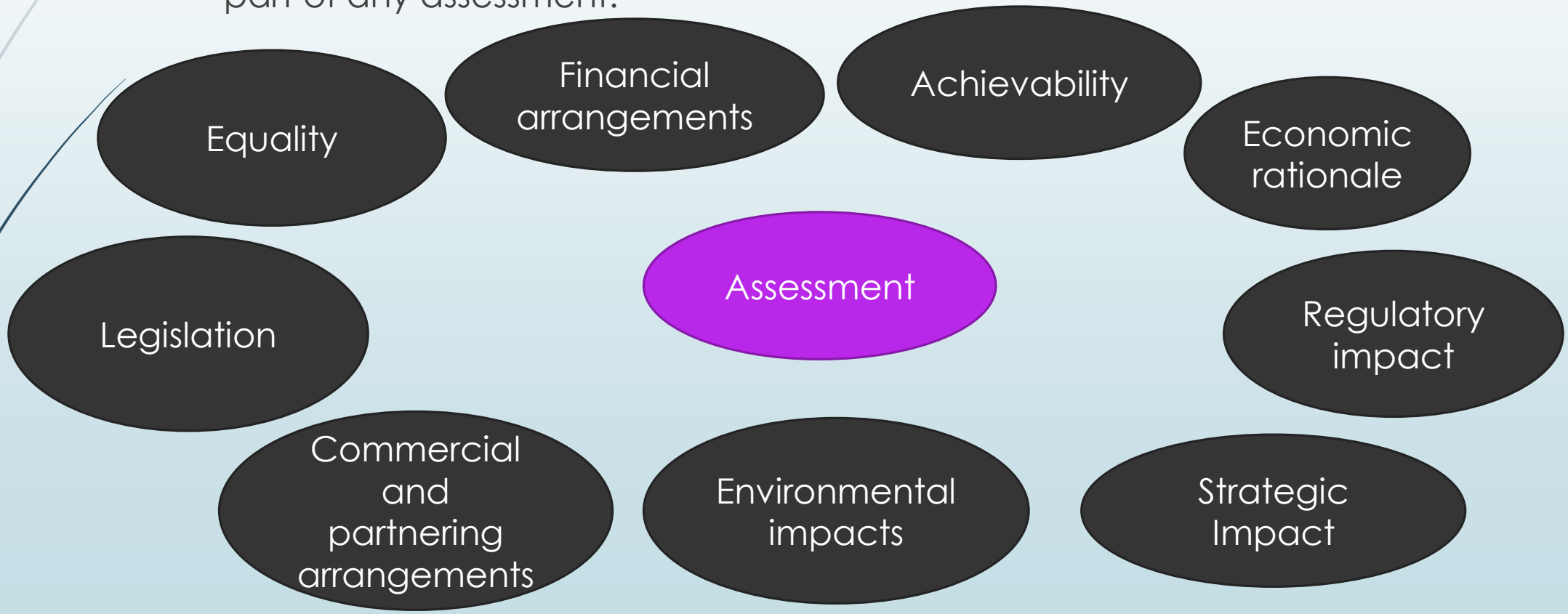
- **Pareto improving investments** (investments that make at least someone better off and no one worse off) – too restrictive.
- **Kaldor-Hicks improving investments** (investments made only if those that are made better off could in principle compensate those that are made worse off)
- **Difficult to quantify the future benefits** of a public utility investment. This means that frequently governments tend to take the benefits as given without quantifying them and concern themselves only with the **minimization of costs**.

Applying investment appraisal techniques to public utilities

- Capital-intensive investments are appraised by taking into account the **whole-life costs** across the **complete life cycle** of the investment as there may be significant **termination/decommissioning costs**.
- In the public sector, it is common practice to identify the option with the **lowest whole-life cost** as the option that offers the **best value for money (VfM)**.
- An alternative is to carry out a **Cost/Benefit Analysis** of the proposed investment, identifying all the costs and benefits whether they be **social or economic** and giving them a **monetary value**. For example in the case of a wastewater treatment plant, not only would we quantify the number of new connections to the sewage system, but also the reduction in environmental damage, the improved health outcomes, etc.

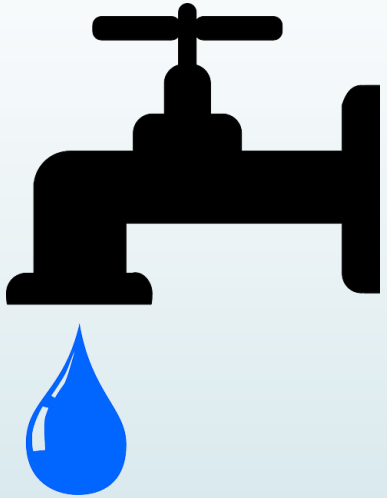
Applying investment appraisal techniques to public utilities

- There is a wide range of generic issues that may need to be considered as part of any assessment.



Investment appraisal in public sector – indirect benefits

Pay taxes to help build water treatment plant



Benefit from clean water

Less hospital admissions

Less environmental damage from misuse of scarce resources

Improved equality

Less taxes

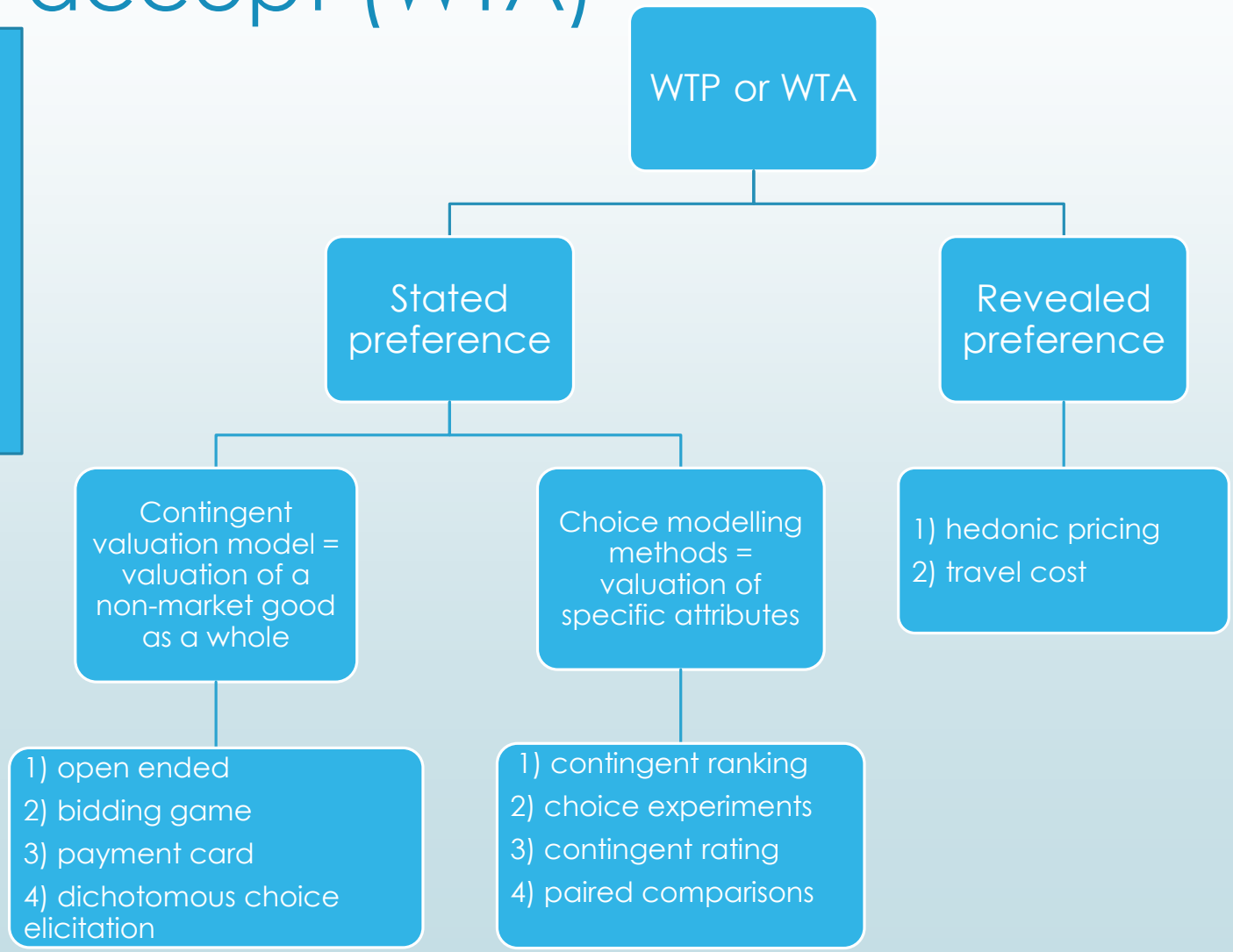
Investment appraisal in public sector – how to calculate indirect benefits

- If indirect benefits can be expressed as a monetary value, use monetary value
- If indirect benefits can't be expressed as a monetary value use:

- Willingness – to pay
- Multi – criteria analysis - weight and score the performance criteria (critical success factors) of the benefits

Willingness –to- Pay (WTP) or willingness-to-accept (WTA)

WTP is the maximum amount of money an individual is willing to give up to receive a good



WTA is the minimum amount of money an individual is willing to be compensated for foregoing a good

Multi-criteria analysis or multi-criteria decision analysis (MCA or MCDA)

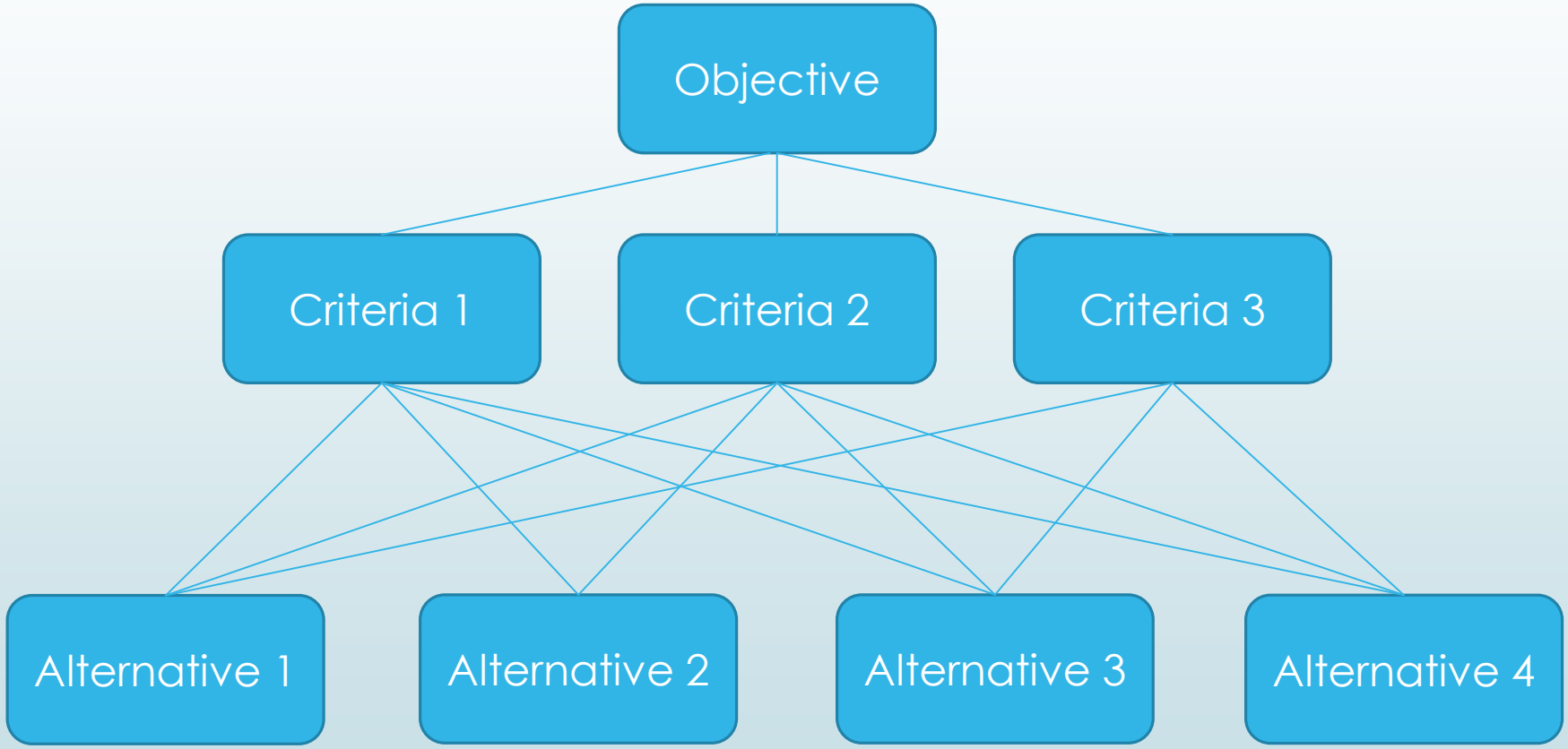
- MCA establishes preferences between options by referring to an explicit **set of objectives/focus** that the decision making body has identified and for which it has established **measurable criteria** to assess the extent to which the objectives are achieved in relation to the different **options/alternatives**
- MCA is a way of looking at **complex problems** that are characterized by any mixture of monetary and non-monetary **objectives**, of breaking the problem into more manageable pieces to **allow data and judgements** to be brought to bear on the pieces, and then of reassembling the pieces to present a coherent overall picture to decision makers.

MCA – key steps

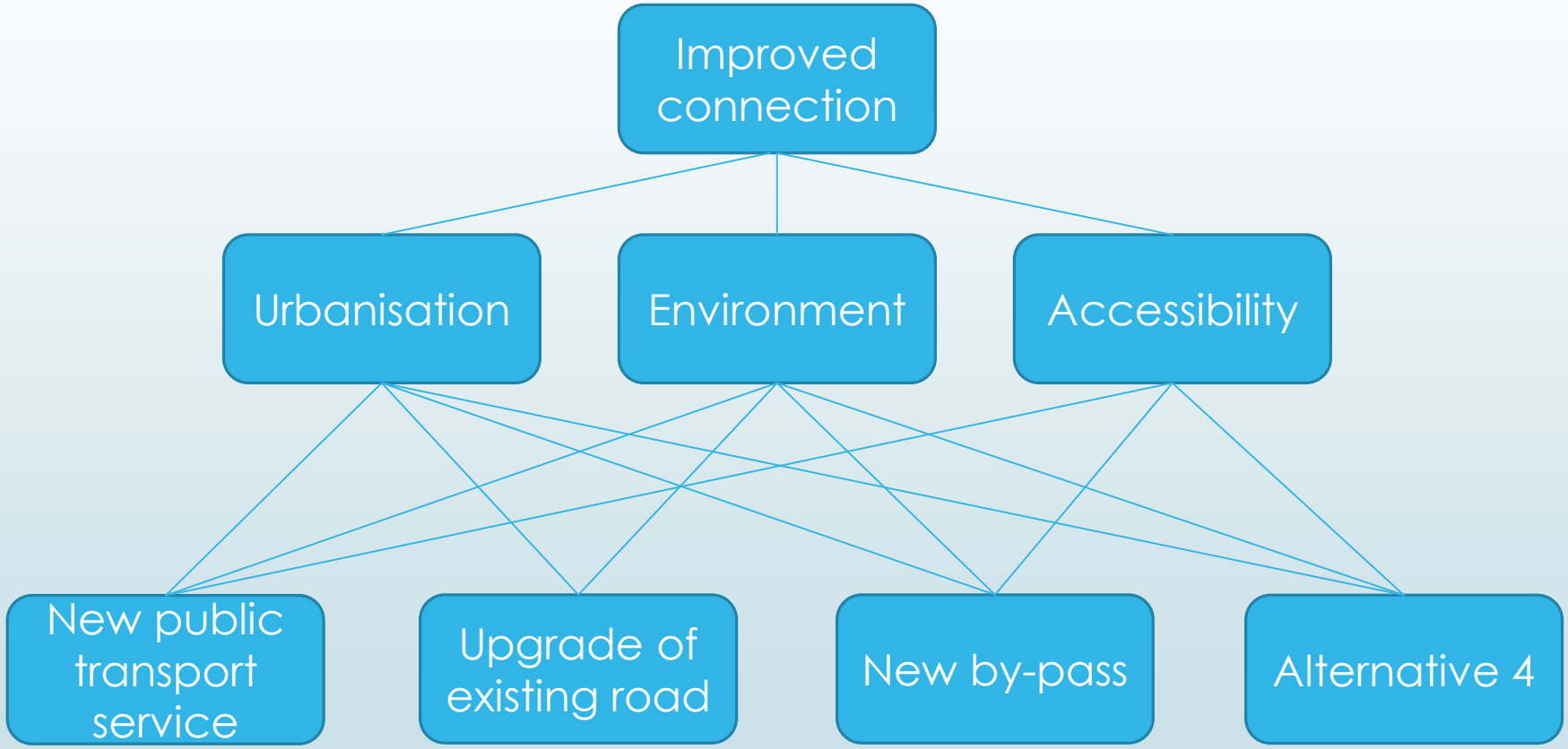
1. Establish the decision context. What are the aims of the MCA, and who are the decision makers and other key players?
2. Identify the options.
3. Identify the objectives and criteria that reflect the value associated with the consequences of each option.
4. Describe the expected performance of each option against the criteria. (If the analysis is to include steps 5 and 6, also 'score' the options, i.e. assess the value associated with the consequences of each option.)
5. 'Weighting'. Assign weights for each of the criteria to reflect their relative importance to the decision.
6. Combine the weights and scores for each of the options to derive and overall value.
7. Examine the results.
8. Conduct a sensitivity analysis of the results to changes in scores or weights.

Source – Multi-criteria analysis: a manual, Department for Communities and Local Government, January 2009

MCA – Analytical Hierarchy Process



MCA – Analytical Hierarchy Process



Applying investment appraisal techniques to public utilities

- For public utilities which are government held, some adjustments have to be made to the cash flows and the discounting rate. In particular:
- Cash flows must not be netted for tax as governments do not pay tax. So all cash flows should be considered gross of tax => **pre – tax cash flows**
- In the calculation of the discount rate, kd should be considered gross of tax too, as the interest payments are not tax deductible for governments => **pre –tax cost of debt**

Thank you for listening! Any questions?

Investment choices

Sources

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NPV – Present value table

n/i	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	20.0%
1	0.99010	0.98522	0.98039	0.97561	0.97087	0.96618	0.96154	0.95694	0.95238	0.94787	0.94340	0.93458	0.92593	0.91743	0.90909	0.90090	0.89286	0.83333
2	0.98030	0.97066	0.96117	0.95181	0.94260	0.93351	0.92456	0.91573	0.90703	0.89845	0.89000	0.87344	0.85734	0.84168	0.82645	0.81162	0.79719	0.69444
3	0.97059	0.95632	0.94232	0.92860	0.91514	0.90194	0.88900	0.87630	0.86384	0.85161	0.83962	0.81630	0.79383	0.77218	0.75131	0.73119	0.71178	0.57870
4	0.96098	0.94218	0.92385	0.90595	0.88849	0.87144	0.85480	0.83856	0.82270	0.80722	0.79209	0.76290	0.73503	0.70843	0.68301	0.65873	0.63552	0.48225
5	0.95147	0.92826	0.90573	0.88385	0.86261	0.84197	0.82193	0.80245	0.78353	0.76513	0.74726	0.71299	0.68058	0.64993	0.62092	0.59345	0.56743	0.40188
6	0.94205	0.91454	0.88797	0.86230	0.83748	0.81350	0.79031	0.76790	0.74622	0.72525	0.70496	0.66634	0.63017	0.59627	0.56447	0.53464	0.50663	0.33490
7	0.93272	0.90103	0.87056	0.84127	0.81309	0.78599	0.75992	0.73483	0.71068	0.68744	0.66506	0.62275	0.58349	0.54703	0.51316	0.48166	0.45235	0.27908
8	0.92348	0.88771	0.85349	0.82075	0.78941	0.75941	0.73069	0.70319	0.67684	0.65160	0.62741	0.58201	0.54027	0.50187	0.46651	0.43393	0.40388	0.23257
9	0.91434	0.87459	0.83676	0.80073	0.76642	0.73373	0.70259	0.67290	0.64461	0.61763	0.59190	0.54393	0.50025	0.46043	0.42410	0.39092	0.36061	0.19381
10	0.90529	0.86167	0.82035	0.78120	0.74409	0.70892	0.67556	0.64393	0.61391	0.58543	0.55839	0.50835	0.46319	0.42241	0.38554	0.35218	0.32197	0.16151
11	0.89632	0.84893	0.80426	0.76214	0.72242	0.68495	0.64958	0.61620	0.58468	0.55491	0.52679	0.47509	0.42888	0.38753	0.35049	0.31728	0.28748	0.13459
12	0.88745	0.83639	0.78849	0.74356	0.70138	0.66178	0.62460	0.58966	0.55684	0.52598	0.49697	0.44401	0.39711	0.35553	0.31863	0.28584	0.25668	0.11216
13	0.87866	0.82403	0.77303	0.72542	0.68095	0.63940	0.60057	0.56427	0.53032	0.49856	0.46884	0.41496	0.36770	0.32618	0.28966	0.25751	0.22917	0.09346
14	0.86996	0.81185	0.75788	0.70773	0.66112	0.61778	0.57748	0.53997	0.50507	0.47257	0.44230	0.38782	0.34046	0.29925	0.26333	0.23199	0.20462	0.07789
15	0.86135	0.79985	0.74301	0.69047	0.64186	0.59689	0.55526	0.51672	0.48102	0.44793	0.41727	0.36245	0.31524	0.27454	0.23939	0.20900	0.18270	0.06491
16	0.85282	0.78803	0.72845	0.67362	0.62317	0.57671	0.53391	0.49447	0.45811	0.42458	0.39365	0.33873	0.29189	0.25187	0.21763	0.18829	0.16312	0.05409
17	0.84438	0.77639	0.71416	0.65720	0.60502	0.55720	0.51337	0.47318	0.43630	0.40245	0.37136	0.31657	0.27027	0.23107	0.19784	0.16963	0.14564	0.04507
18	0.83602	0.76491	0.70016	0.64117	0.58739	0.53836	0.49363	0.45280	0.41552	0.38147	0.35034	0.29586	0.25025	0.21199	0.17986	0.15282	0.13004	0.03756
19	0.82774	0.75361	0.68643	0.62553	0.57029	0.52016	0.47464	0.43330	0.39573	0.36158	0.33051	0.27651	0.23171	0.19449	0.16351	0.13768	0.11611	0.03130
20	0.81954	0.74247	0.67297	0.61027	0.55368	0.50257	0.45639	0.41464	0.37689	0.34273	0.31180	0.25842	0.21455	0.17843	0.14864	0.12403	0.10367	0.02608
21	0.81143	0.73150	0.65978	0.59539	0.53755	0.48557	0.43883	0.39679	0.35894	0.32486	0.29416	0.24151	0.19866	0.16370	0.13513	0.11174	0.09256	0.02174
24	0.78757	0.69954	0.62172	0.55288	0.49193	0.43796	0.39012	0.34770	0.31007	0.27666	0.24698	0.19715	0.15770	0.12640	0.10153	0.08170	0.06588	0.01258
25	0.77977	0.68921	0.60953	0.53939	0.47761	0.42315	0.37512	0.33273	0.29530	0.26223	0.23300	0.18425	0.14602	0.11597	0.09230	0.07361	0.05882	0.01048
28	0.75684	0.65910	0.57437	0.50088	0.43708	0.38165	0.33348	0.29157	0.25509	0.22332	0.19563	0.15040	0.11591	0.08955	0.06934	0.05382	0.04187	0.00607
29	0.74934	0.64936	0.56311	0.48866	0.42435	0.36875	0.32065	0.27902	0.24295	0.21168	0.18456	0.14056	0.10733	0.08215	0.06304	0.04849	0.03738	0.00506
30	0.74192	0.63976	0.55207	0.47674	0.41199	0.35628	0.30832	0.26700	0.23138	0.20064	0.17411	0.13137	0.09938	0.07537	0.05731	0.04368	0.03338	0.00421
31	0.73458	0.63031	0.54125	0.46511	0.39999	0.34423	0.29646	0.25550	0.22036	0.19018	0.16425	0.12277	0.09202	0.06915	0.05210	0.03935	0.02980	0.00351
40	0.67165	0.55126	0.45289	0.37243	0.30656	0.25257	0.20829	0.17193	0.14205	0.11746	0.09722	0.06678	0.04603	0.03184	0.02209	0.01538	0.01075	0.00068